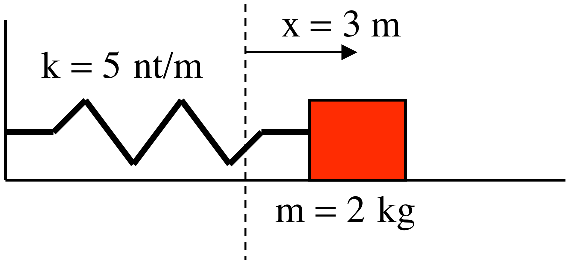
Problem 13.31

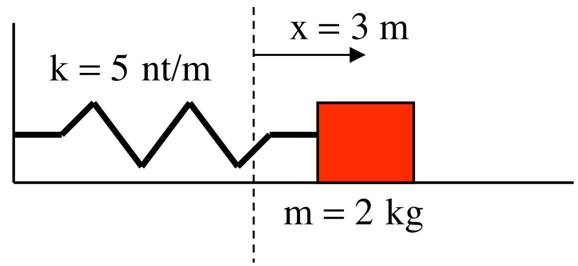
When a 2 kg mass is attached to a spring with a spring constant of 5 nt/m. It is elongated a distance 3 meters from its equilibrium position and releases at t=0.



a.) What is the force 3.5 seconds after release?

b.) Through how many cycles does the body oscillate in 3.5 seconds?

When a 2 kg mass is attached to a spring with a spring constant of 5 nt/m. It is elongated a distance 3 meters from its equilibrium position and releases at t=0.



a.) What is the force 3.5 seconds after release?

To get the force, you need the acceleration. How do you get the argument for the sine function? We know that for a spring. The amplitude is 3 meters and the angular frequency is:

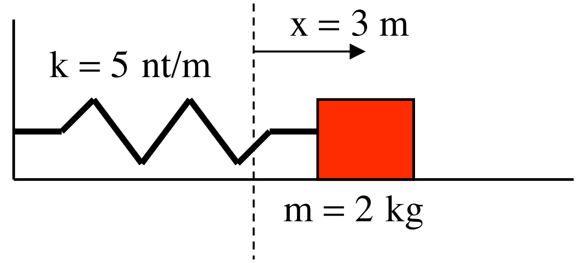
$$\omega = \left(\frac{k}{m}\right)^{1/2}$$
$$= \left(\frac{5\ \text{nt/m}}{2\ \text{kg}}\right)^{1/2}$$
$$= 1.58\ \text{rad/sec}$$

Then the acceleration function looks like:

$$a = -\omega^2 A \sin(\omega t + \phi)$$
$$= -(1.58\ \text{rad/sec})^2 (3\ \text{m}) \sin((1.58\ \text{rad/sec})t + \phi)$$
$$= -7.48 \sin((1.58\ \text{rad/sec})t + \phi)$$

All we need is the phase shift ϕ. I'll do this the formal way. That way you can see how it is done in general.

Start with the *position versus time* relationship in both algebraic and numeric form (to be complete):



$$x = A\sin(\omega t + \phi) \qquad\qquad x = (3\text{ m})\sin(1.58t + \phi)$$

The trick is to put in the information you know at a time you know. In this case, and in all cases, that will be what is happening at t=0. Specifically, at t=0, the position is at the maximum (A=3 meters). Using this information in both an algebraic and numerical version of the relationship, we get:

$$\begin{aligned} &x = A\sin(\omega t + \phi) \\ &\Rightarrow \quad A = A\sin(\omega(0) + \phi) \\ &\Rightarrow \quad 1 = \sin\phi \\ &\Rightarrow \quad \phi = \sin^{-1}(1) \\ &\Rightarrow \quad \phi = 1.57 \end{aligned}$$

$$\begin{aligned} &x = (3\text{ m})\sin(1.58t + \phi) \\ &\Rightarrow \quad (3\text{ m}) = (3\text{ m})\sin(1.58(0) + \phi) \\ &\Rightarrow \quad 1 = \sin\phi \\ &\Rightarrow \quad \phi = \sin^{-1}(1) \\ &\Rightarrow \quad \phi = 1.57 \end{aligned}$$

This means we can now write the acceleration expression as:

$$a = -7.48\sin\left((1.58\text{ rad/sec})t + (1.57\text{ rad})\right)$$

That means the force at 3.5 seconds will be::

$$\begin{aligned} F = ma &= -(2\text{ kg})\left(7.48\sin\left((1.58\text{ rad/sec})t + (1.57\text{ rad})\right)\right) \\ &= -(2\text{ kg})\left(7.48\sin\left((1.58\text{ rad/sec})(3.5\text{ sec}) + (1.57\text{ rad})\right)\right) \\ &= -10.9\text{ nts} \end{aligned}$$

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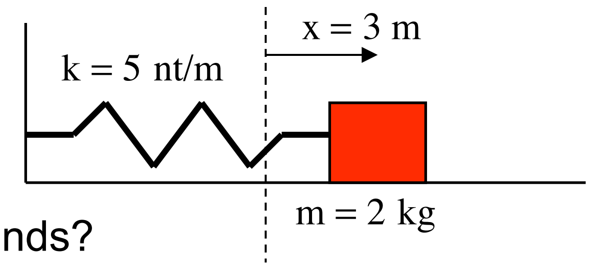

b.) Through how many cycles does the body oscillate in 3.5 seconds?

We know the angular frequency. From it we can get the frequency and with that, the solutions.

$$\omega = 1.58 \text{ rad/sec and } \omega = 2\pi\nu, \text{ so}: \quad (1.58 \text{ rad/sec}) = 2\pi\nu$$

$$\Rightarrow \quad \nu = \frac{(1.58 \text{ rad/sec})}{2\pi}$$

$$\Rightarrow \quad \nu = .25 \text{ cycles/sec}$$

Oscillating at .25 cycles per second for 3.5 seconds yields .88 cycles swept through.